Chapter 3: The World of Numbers
Mathematics Notes

Exercise Set 3.1

Question 1

Solution:
Let x be the number of copper ingots left with.
Ratio is $\frac{15 \text{ ingots}}{2 \text{ bags}}$.
For 12 bags:
$x = 12 \times \frac{15}{2} = 6 \times 15 = 90$.
He will leave with 90 copper ingots.

Formula / Hints:
Unitary Method / Direct Proportion:
$\text{Ingots} = \text{Bags} \times \left(\frac{\text{Ingots}}{\text{Bag}}\right)$

Question 2

Solution:
The sequence is $11, 13, 17, 19$.
Common property: They are consecutive prime numbers.
The next three prime numbers are $23, 29, 31$.

Hints:
Prime numbers only have factors 1 and themselves.

Question 3

Solution:
Natural numbers are not closed under subtraction.
Example 1: $3 - 5 = -2 \notin \mathbb{N}$
Example 2: $10 - 10 = 0 \notin \mathbb{N}$
Since -2 and 0 are not natural numbers, closure fails.

Hints:
Closure property means the output must belong to the same number set as the inputs.

Question 4*

Solution:
A hand has 4 fingers with 3 joints each $\implies 4 \times 3 = 12$ joints.
The thumb is used as the pointer to count them.
One full hand can count up to 12.
This directly forms the base unit for a base-12 (duodecimal) system.

Hints:
Counting finger segments instead of whole fingers allows counting up to 12 on one hand.

Exercise Set 3.2

Question 1

Solution:
Initial Temperature = 4°C.
Drop = 15°C.
Midnight Temp = $4 - 15 = -11$°C.

Hints:
Final = Initial − Drop

Question 2

Solution:
Loan (debt) = -850.
Profit = $+1200$.
Loss = -450.
Equation: $S = -850 + 1200 - 450$.
Calculation: $S = 350 - 450 = -100$.
Final standing: Debt of 100.

Hints:
Debts/losses are negative numbers; profits/gains are positive numbers.

Question 3

Solution:
(i) $(-12) \times 5 = -60$
(ii) $(-8) \times (-7) = 56$
(iii) $0 - (-14) = 0 + 14 = 14$
(iv) $(-20) \div 4 = -5$

Formula:
$(-) \times (+) = (-)$
$(-) \times (-) = (+)$
$-(-x) = +x$

Question 4

Solution:
Example: If you owe someone 10 (standing at -10), and that debt of 10 is taken away (subtracted), it is exactly equivalent to receiving 10 cash (adding $+10$).
Mathematically: $10 - (-5) = 10 + 5 = 15$.

Hints:
Subtracting a negative removes a baseline reduction, shifting value upwards.

End-of-Chapter Exercises

Question 1

Solution:
(i) $\frac{3}{50} = \frac{3\times2}{50\times2} = \frac{6}{100} = 0.06$ (Terminating)
(ii) $\frac{2}{9} = 0.2222... = 0.\overline{2}$ (Non-terminating repeating)

Hints:
Terminating if denominator prime factors contain only 2 or 5.

Question 2*

Solution:
Let $\sqrt{5} = \frac{p}{q}$ where $\gcd(p,q) = 1$.
Squaring: $5 = \frac{p^2}{q^2} \implies p^2 = 5q^2 \implies 5 \text{ divides } p^2 \implies 5 \text{ divides } p$.
Let $p = 5k \implies (5k)^2 = 5q^2 \implies 25k^2 = 5q^2 \implies q^2 = 5k^2 \implies 5 \text{ divides } q$.
This contradicts $\gcd(p,q) = 1$. Thus, $\sqrt{5}$ is irrational.

Hints:
Proof by contradiction.

Question 3

Solution:
(i) $12.6 = \frac{126}{10} = \frac{63}{5}$
(ii) $0.0120 = \frac{120}{10000} = \frac{3}{250}$
(iii) Let $x = 3.\overline{052} \implies 1000x = 3052.\overline{052}$
$999x = 3049 \implies x = \frac{3049}{999}$
(iv) Let $x = 1.2\overline{35} \implies 10x = 12.\overline{35}, 1000x = 1235.\overline{35}$
$990x = 1223 \implies x = \frac{1223}{990}$
(v) $0.\overline{23} = \frac{23}{99}$ (vi) $2.0\overline{5} = \frac{205-20}{90} = \frac{185}{90} = \frac{37}{18}$
(vii) $2.12\overline{5} = \frac{2125-212}{900} = \frac{1913}{900}$
(viii) $3.12\overline{5} = \frac{3125-312}{900} = \frac{2813}{900}$
(ix) $2.\overline{1625} = \frac{21625-2}{9999} = \frac{21623}{9999}$

Formula:
Pure recurring fraction conversion layout shortcut:
$x = \frac{\text{Full Number}-\text{Non-repeating}}{\text{9's followed by 0's}}$

Question 4

Solution:
(i) 0.532: Divide segment $[0,1]$ into 10 parts, locate past 0.5, subdivide further.
(ii) $1.1\overline{5}$: Located between 1.1 and 1.2 on the number line.

Hints:
Successive magnification maps decimal locations sequentially.

Question 5

Solution:
Multiply numerator and denominator of 3 and 4 by $6 + 1 = 7$:
$3 = \frac{21}{7}, \quad 4 = \frac{28}{7}$
Six numbers: $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$

Hints:
To find n rational numbers, scale by $(n+1)$.

Question 6

Solution:
Convert denominators to common base ($5 \times 6 = 30$):
$\frac{2}{5} = \frac{12}{30}, \quad \frac{3}{5} = \frac{18}{30}$
Five numbers: $\frac{13}{30}, \frac{14}{30}, \frac{15}{30}, \frac{16}{30}, \frac{17}{30}$

Hints:
Equate denominators to uncover hidden internal integer steps.

Question 7

Solution:
Make denominators common LCM(6,5)=30:
$\frac{1}{6} = \frac{5}{30} = \frac{30}{180}, \quad \frac{2}{5} = \frac{12}{30} = \frac{72}{180}$
Five numbers: $\frac{31}{180}, \frac{32}{180}, \frac{33}{180}, \frac{34}{180}, \frac{35}{180}$

Hints:
Find LCM first, scale up further if wider integer span is needed.

Question 8

Solution:
$\frac{x}{3} + \frac{x}{5} = \frac{16}{15} \implies \frac{5x+3x}{15} = \frac{16}{15}$
$8x = 16 \implies x = 2.$

Formula:
Fraction addition via LCM.

Question 9

Solution:
Given $a + \frac{1}{b} = 0 \implies a = -\frac{1}{b} \implies ab = -1.$
Since $ab = -1$, the product ab is strictly **negative**.

Hints:
Non-zero inverses carry the same sign, so their product across a negative inversion must equal -1.

Question 10

Solution:
Let terminating value be $x = \frac{p}{10^4} = \frac{p}{2^4 \times 5^4}$.
In lowest terms, common factors are cancelled. Since the final non-zero digit is at the 4th decimal place, the denominator must retain factors of both 2^4 and 5^4 before matching fractions. Thus it is divisible by both 2^4 or 5^4.

Hints:
Terminating limits match prime factorization characteristics of the decimal base.

Question 11

Solution:
$\frac{18}{125} = \frac{18}{5^3}$. Since denominator prime factors contain only 5, it is **terminating**.
Number of decimal places = highest exponent of 5 = 3 places.

Hints:
Decimal places equal the max power index value when base factors are isolated.

Question 12

Solution:
Denominator $= 2^3 \times 5^1$.
Highest exponent between 2 and 5 is 3.
Therefore, the decimal expansion will have exactly **3** decimal places.

Hints:
The highest power index dictates the termination limit.

Question 13*

Solution:
$a = \frac{7}{12} = \frac{35}{60}, \quad b = \frac{5}{6} = \frac{50}{60}$
Let common denominator $m = 60$. Then $k_1 = 35, k_2 = 50$.
$k_2 - k_1 = 50 - 35 = 15 > 6$. Valid.
Five distinct numbers: $\frac{36}{60}, \frac{37}{60}, \frac{38}{60}, \frac{39}{60}, \frac{40}{60}$
Condition $k_2 - k_1 > n + 1$ guarantees there are at least n whole integers safely between the two scaled values.

Hints:
The gap between integers must exceed the requested count bounds to secure room.

Question 14*

Solution:
$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$
Substitute $x + y + z = 0$ and $xy + yz + zx = 0$:
$0^2 = x^2 + y^2 + z^2 + 2(0) \implies x^2 + y^2 + z^2 = 0$.
Since squares of real numbers cannot be negative, $x^2 = 0, y^2 = 0, z^2 = 0 \implies x = 0, y = 0, z = 0$.

Formula:
Algebraic identity:
$(x + y + z)^2 = \sum x^2 + 2 \sum xy$

Question 15*

Solution:
We need to show $a < \frac{a+b}{2} < b$ given $a < b$:
Add a to both sides: $2a < a + b \implies a < \frac{a+b}{2}$.
Add b to both sides: $a + b < 2b \implies \frac{a+b}{2} < b$.
Therefore, $\frac{a+b}{2}$ lies between a and b.

Hints:
The mean point always naturally splits an ordered line interval.

Question 16

Solution:
Using the Pythagorean theorem sequentially:
Hypotenuse 1 $= \sqrt{1^2 + 1^2} = \sqrt{2}$
Hypotenuse 2 $= \sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{2 + 1} = \sqrt{3}$
Hypotenuse 3 $= \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3 + 1} = \sqrt{4} = 2$
The sequential lengths are $\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \ldots, \sqrt{17}$

Formula:
Pythagorean Theorem:
$h = \sqrt{\text{base}^2 + \text{perpendicular}^2}$
