Chapter 2: Polynomials
Complete Board PYQs Question Bank

Exercise Set 2.1 (NCERT Core & Board Questions)

Question 1 (i)

Solution:
Given: $f(x) = x^2 - 2x - 8$.
By splitting the middle term:
$x^2 - 4x + 2x - 8 = 0 \implies x(x-4) + 2(x-4) = 0$
$\implies (x-4)(x+2) = 0 \implies \alpha = 4, \beta = -2$.
Verification:
$\alpha + \beta = 4 + (-2) = 2 = -\frac{b}{a} = -\frac{-2}{1} = 2$.
$\alpha\beta = 4 \times (-2) = -8 = \frac{c}{a} = \frac{-8}{1} = -8$.

Formula / Hints:
Relationship:
Sum of zeros: $\alpha + \beta = -\frac{b}{a}$
Product of zeros: $\alpha\beta = \frac{c}{a}$

Question 1 (ii)

Solution:
Given: $g(s) = 4s^2 - 4s + 1$.
$4s^2 - 2s - 2s + 1 = 0 \implies 2s(2s-1) - 1(2s-1) = 0$
$\implies (2s-1)^2 = 0 \implies \alpha = \frac{1}{2}, \beta = \frac{1}{2}$.
Verification:
$\alpha + \beta = \frac{1}{2} + \frac{1}{2} = 1 = -\frac{-4}{4} = 1$.
$\alpha\beta = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{1}{4}$.

Formula / Hints:
Perfect square trinomial:
$4s^2 - 4s + 1 = (2s-1)^2$.

Question 1 (iii)

Solution:
Given: $h(t) = t^2 - 15$.
$t^2 - (\sqrt{15})^2 = 0 \implies (t - \sqrt{15})(t + \sqrt{15}) = 0$
$\implies \alpha = \sqrt{15}, \beta = -\sqrt{15}$.
Verification:
$\alpha + \beta = \sqrt{15} - \sqrt{15} = 0 = -\frac{0}{1} = 0$.
$\alpha\beta = \sqrt{15}(-\sqrt{15}) = -15 = \frac{-15}{1} = -15$.

Formula / Hints:
Difference of squares:
$a^2 - b^2 = (a-b)(a+b)$.

Question 1 (iv)

Solution:
Given: $f(x) = 6x^2 - 3 - 7x \implies 6x^2 - 7x - 3 = 0$.
$6x^2 - 9x + 2x - 3 = 0 \implies 3x(2x-3) + 1(2x-3) = 0$
$\implies (2x-3)(3x+1) = 0 \implies \alpha = \frac{3}{2}, \beta = -\frac{1}{3}$.
Verification:
$\alpha + \beta = \frac{3}{2} - \frac{1}{3} = \frac{7}{6} = -\frac{-7}{6} = \frac{7}{6}$.
$\alpha\beta = \frac{3}{2}\left(-\frac{1}{3}\right) = -\frac{1}{2} = \frac{-3}{6} = -\frac{1}{2}$.

Formula / Hints:
Rearrange into standard form $ax^2 + bx + c = 0$ first.

Question 1 (v)

Solution:
Given: $q(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3} \implies 21y^2 - 11y - 2 = 0.$
$21y^2 - 14y + 3y - 2 = 0 \implies 7y(3y-2) + 1(3y-2) = 0$
$\implies (3y-2)(7y+1) = 0 \implies \alpha = \frac{2}{3}, \beta = -\frac{1}{7}.$
Verification:
$\alpha + \beta = \frac{2}{3} - \frac{1}{7} = \frac{11}{21} = -\frac{-11/3}{7} = \frac{11}{21}.$
$\alpha\beta = \frac{2}{3}\left(-\frac{1}{7}\right) = -\frac{2}{21} = \frac{-2/3}{7} = -\frac{2}{21}.$

Formula / Hints:
Multiply through by the denominator to clean up factors.

Question 1 (vii)

Solution:
Given: $p(x) = x^2 + \frac{4}{3}x - \frac{4}{3} \implies 3x^2 + 4x - 4 = 0.$
$3x^2 + 6x - 2x - 4 = 0 \implies 3x(x+2) - 2(x+2) = 0$
$\implies (x+2)(3x-2) = 0 \implies \alpha = -2, \beta = \frac{2}{3}.$
Verification:
$\alpha + \beta = -2 + \frac{2}{3} = -\frac{4}{3} = -\frac{4/3}{1} = -\frac{4}{3}.$
$\alpha\beta = -2 \times \frac{2}{3} = -\frac{4}{3} = \frac{-4/3}{1} = -\frac{4}{3}.$

Formula / Hints:
Standard relationship checking steps.

Question 7 (ix)

Solution:
Given: $f(x) = 4x^2 + 4x - 3 = 0.$
$4x^2 + 6x - 2x - 3 = 0 \implies 2x(2x+3) - 1(2x+3) = 0$
$\implies (2x+3)(2x-1) = 0 \implies \alpha = -\frac{3}{2}, \beta = \frac{1}{2}.$
Verification:
$\alpha + \beta = -\frac{3}{2} + \frac{1}{2} = -1 = -\frac{4}{4} = -1.$
$\alpha\beta = -\frac{3}{2} \times \frac{1}{2} = -\frac{3}{4} = \frac{-3}{4}.$

Formula / Hints:
Middle term split rule.

Question 14

Solution:
Given α, β are roots of $px^2 + qx + 1$.
$\alpha + \beta = -\frac{q}{p}$, $\alpha\beta = \frac{1}{p}$.
New roots: $S' = \frac{2}{\alpha} + \frac{2}{\beta} = \frac{2(\alpha+\beta)}{\alpha\beta} = \frac{2(-q/p)}{1/p} = -2q.$
New product: $P' = \frac{2}{\alpha} \times \frac{2}{\beta} = \frac{4}{\alpha\beta} = \frac{4}{1/p} = 4p.$
Required Polynomial: $x^2 - S'x + P' = 0 \implies x^2 + 2qx + 4p = 0.$

Formula / Hints:
New polynomial rule:
$x^2 - (\text{Sum})x + \text{Product} = 0.$

Question 15

Solution:
Given α, β are roots of $ax^2 - x + c$.
$\alpha + \beta = \frac{1}{a}$, $\alpha\beta = \frac{c}{a}$.
New roots: $\alpha - 3$ and $\beta - 3$.
$S' = (\alpha - 3) + \beta - 3 = (\alpha + \beta) - 6 = \frac{1}{a} - 6 = \frac{1-6a}{a}$.
$P' = (\alpha - 3)(\beta - 3) = \alpha\beta - 3(\alpha + \beta) + 9 = \frac{c}{a} - \frac{3}{a} + 9 = \frac{c-3+9a}{a}$.
Required Polynomial: $a[x^2 - S'x + P'] = 0 \implies ax^2 - (1 - 6a)x + (c - 3 + 9a) = 0$.

Formula / Hints:
Expand and group values systematically.

Question 16

Solution:
Given: $7x^2 + 18x - 9 = 0 \implies 7x^2 + 21x - 3x - 9 = 0$
$7x(x + 3) - 3(x + 3) = 0 \implies \alpha = -3, \beta = \frac{3}{7}$.
New roots are twice the original ones: $\alpha' = -6, \beta' = \frac{6}{7}$.
$S' = -6 + \frac{6}{7} = -\frac{36}{7}$, $P' = -6 \times \frac{6}{7} = -\frac{36}{7}$.
Required Polynomial: $7[x^2 - S'x + P'] = 0 \implies 7x^2 + 36x - 36 = 0$.

Formula / Hints:
Find original roots explicitly or scale variable transformations.

Question 18

Solution:
Given: $3x^2 - 4x - 4 = 0 \implies 3x^2 - 6x + 2x - 4 = 0$
$3x(x - 2) + 2(x - 2) = 0 \implies \alpha = 2, \beta = -\frac{2}{3}$.
New roots are one less than each: $\alpha' = 1, \beta' = -\frac{5}{3}$.
$S' = 1 - \frac{5}{3} = -\frac{2}{3}$, $P' = 1 \times \left(-\frac{5}{3}\right) = -\frac{5}{3}$.
Required Polynomial: $3[x^2 - S'x + P'] = 0 \implies 3x^2 + 2x - 5 = 0$.

Formula / Hints:
New roots $\implies (\alpha - 1), (\beta - 1)$.

Question 19

Solution:
Given: $q(x) = 8x^2 - 2x - 3 = 0 \implies 8x^2 - 6x + 4x - 3 = 0$
$2x(4x - 3) + 1(4x - 3) = 0 \implies \alpha = \frac{3}{4}, \beta = -\frac{1}{2}$.
New roots are 2 less than each: $\alpha' = \frac{3}{4} - 2 = -\frac{5}{4}, \beta' = -\frac{1}{2} - 2 = -\frac{5}{2}$.
$S' = -\frac{5}{4} - \frac{5}{2} = -\frac{15}{4}$, $P' = \left(-\frac{5}{4}\right)\left(-\frac{5}{2}\right) = \frac{25}{8}$.
Required Polynomial: $8[x^2 - S'x + P'] = 0 \implies 8x^2 + 30x + 25 = 0$.

Formula / Hints:
Scale fraction operations cleanly to match denominator bases.

Question 20

Solution:
Given: $r(x) = 4x^2 + 3x - 1 = 0 \implies 4x^2 + 4x - x - 1 = 0$
$4x(x + 1) - 1(x + 1) = 0 \implies \alpha = \frac{1}{4}, \beta = -1$.
New roots are reciprocal: $\alpha' = 4, \beta' = -1$.
$S' = 4 + (-1) = 3$, $P' = 4 \times (-1) = -4$.
Required Polynomial: $x^2 - 3x - 4 = 0$.

Formula / Hints:
Reciprocal root transformation turns $ax^2 + bx + c$ into $cx^2 + bx + a$.

Exercise Set 2.2 (CBSE Board Questions)

Question 7

Solution:
Given 4 is a zero of $x^3 - 3x^2 - 10x + 24 = 0$.
By synthetic/long division by $(x - 4)$:
$x^3 - 3x^2 - 10x + 24 = (x - 4)(x^2 + x - 6) = 0$.
Solve the quadratic component: $x^2 + 3x - 2x - 6 = 0$
$\implies x(x+3) - 2(x+3) = 0 \implies (x+3)(x-2) = 0 \implies$
$x = 2, -3$.
The other two zeros are 2 and -3.

Formula / Hints:
Cubic division rules reduce polynomials down cleanly.

Exercise Set 2.3 (CBSE Board Questions)

Question 3

Solution:
Given roots are -2 and -1. Product factor $= (x+2)(x+1) = x^2 + 3x + 2$.
Divide $2x^4 + x^3 - 14x^2 - 19x - 6$ by $(x^2 + 3x + 2)$:
We obtain the quotient $(2x^2 - 5x - 3)$.
Factor the quotient: $2x^2 - 6x + x - 3 = 0 \implies 2x(x - 3) + 1(x - 3) = 0$
$\implies (x - 3)(2x + 1) = 0 \implies x = 3, -\frac{1}{2}$.
All zeros are $-2, -1, 3, -\frac{1}{2}$.

Formula / Hints:
Bi-quadratic tracking reduction layout.

Question 4

Solution:
Given roots are 2 and -2. Product factor $= (x - 2)(x + 2) = x^2 - 4$.
Divide $x^4 + x^3 - 34x^2 - 4x + 120$ by $(x^2 - 4)$:
We obtain the quotient $(x^2 + x - 30)$.
Factor the quotient: $x^2 + 6x - 5x - 30 = 0 \implies (x + 6)(x - 5) = 0 \implies x = -6, 5$.
All zeros are $2, -2, 5, -6$.

Formula / Hints:
Polynomial long division method application.

Question 5

Solution:
For $3x^4 - 9x^3 + x^2 + 15x + k$ to be divisible by $(3x^2 - 5)$:
By long division, tracking the remainder matching rules yields:
The remainder simplifies down to exactly $k + 5 = 0 \implies k = -5$.

Formula / Hints:
Completely divisible means remainder must be identically zero.

Question 6

Solution:
Given zeros are $-\sqrt{3}$ and $\sqrt{3} \implies (x + \sqrt{3})(x - \sqrt{3}) = x^2 - 3$.
Divide $2x^3 + x^2 - 6x - 3$ by $(x^2 - 3)$:
We obtain the quotient $(2x + 1) = 0 \implies x = -\frac{1}{2}$.
All zeros are $-\sqrt{3}, \sqrt{3}, -\frac{1}{2}$.

Formula / Hints:
Isolate linear roots from remainder balances.

Question 7

Solution:
Given zeros are $-\sqrt{2}$ and $\sqrt{2} \implies (x+\sqrt{2})(x-\sqrt{2}) = x^2 - 2$.
Divide $x^3 + 3x^2 - 2x - 6$ by $(x^2 - 2)$:
We obtain the quotient $(x+3) = 0 \implies x = -3$.
All zeros are $-\sqrt{2}, \sqrt{2}, -3$.

Formula / Hints:
Factor pairs cancel symmetric irrationals.

Question 9

Solution:
Given zeros are $\sqrt{2}$ and $-\sqrt{2} \implies (x-\sqrt{2})(x+\sqrt{2}) = x^2 - 2$.
Divide $2x^4 + 7x^3 - 19x^2 - 14x + 30$ by $(x^2 - 2)$:
We obtain the quotient $(2x^2 + 7x - 15)$.
Factor the quotient: $2x^2 + 10x - 3x - 15 = 0 \implies 2x(x+5) - 3(x+5) = 0$
$\implies (x+5)(2x-3) = 0 \implies x = -5, \frac{3}{2}$.
All zeros are $\sqrt{2}, -\sqrt{2}, -5, \frac{3}{2}$.

Formula / Hints:
Perform systemic multi-stage trinomial solutions.