Chapter 2: Introduction to Linear Polynomials
Mathematics Notes

Exercise Set 2.5

Question 1

Solution:
Given $y = ax + b$.
At $x = 10$, $y = 400 \implies 10a + b = 400 \quad (1)$
At $x = 14$, $y = 500 \implies 14a + b = 500 \quad (2)$
Subtracting (1) from (2): $4a = 100 \implies a = 25$
Substitute $a = 25$ in (1): $10(25) + b = 400 \implies b = 150$.
Values are: $a = 25, b = 150$.

Formula / Hints:
Linear system setup.
y = total bill, x = modules accessed.

Question 2

Solution:
Given $y = ax + b$.
At $x = 10$, $y = 800 \implies 10a + b = 800 \quad (1)$
At $x = 15$, $y = 1100 \implies 15a + b = 1100 \quad (2)$
Subtracting (1) from (2): $5a = 300 \implies a = 60$
Substitute $a = 60$ in (1): $10(60) + b = 800 \implies b = 200$.
Values are: $a = 60, b = 200$.

Formula / Hints:
Linear system setup.
y = total bill, x = hours of use.

Question 3

Solution:
Given $°C = a(°F) + b$.
At $°C = 0, °F = 32 \implies 32a + b = 0 \quad (1)$
At $°C = 100, °F = 212 \implies 212a + b = 100 \quad (2)$
Subtracting (1) from (2): $180a = 100 \implies a = \frac{5}{9}$
Substitute a in (1): $32\left(\frac{5}{9}\right) + b = 0 \implies b = -\frac{160}{9}$.
Linear relation: $°C = \frac{5}{9}(°F - 32)$.

Formula / Hints:
Simultaneous equations for temperature transformation.

End of Chapter Exercise

Question 1

Solution:
A standard cubic polynomial form is:
$p(x) = kx^3 - 7x^2 + cx + d \quad$ (where $k \neq 0$).
An explicit valid example is: $p(x) = x^3 - 7x^2 + x + 1$.

Hints:
Degree 3 means highest power is 3. Coefficient of x^2 is fixed at -7.

Question 2

Solution:
(i) For $p(x) = 5x^2 - 3x + 7$ at $x = 1$:
$p(1) = 5(1)^2 - 3(1) + 7 = 5 - 3 + 7 = 9$.
(ii) For $p(t) = 4t^3 - t^2 + 6$ at $t = a$:
$p(a) = 4a^3 - a^2 + 6$.

Hints:
Polynomial evaluation via direct substitution.

Question 3

Solution:
Let the number be x.
$\frac{5}{2}x + \frac{2}{3} = \frac{-7}{12} \implies \frac{5}{2}x = \frac{-7}{12} - \frac{8}{12} = \frac{-15}{12} = \frac{-5}{4}$
$x = \frac{-5}{4} \times \frac{2}{5} \implies x = -\frac{1}{2}$.

Formula / Hints:
Linear equation formation: $\frac{5}{2}x + \frac{2}{3} = \frac{-7}{12}$

Question 4

Solution:
Let numbers be x and $5x$.
$5x + 21 = 2(x + 21) \implies 5x + 21 = 2x + 42$
$3x = 21 \implies x = 7$. The numbers are 7 and 35.

Hints:
Larger value matches double the smaller value after adding 21.

Question 5

Solution:
Let m be months. Pattern: $A(m) = 800 + 250m$.
(i) After 6 months: $A(6) = 800 + 250(6) = 2300$.
(ii) After 2 years ($m = 24$): $A(24) = 800 + 250(24) = 6800$.

Formula / Hints:
Arithmetic Pattern: $A(m) = \text{Initial} + \text{Rate} \times m$

Question 6*

Solution:
Let units digit be x. Tens digit is $x + 3$.
Original $= 10(x + 3) + x = 11x + 30$.
Interchanged $= 10x + (x + 3) = 11x + 3$.
Sum: $(11x + 30) + (11x + 3) = 143 \implies 22x + 33 = 143$
$22x = 110 \implies x = 5$. Digits are 5 and 8; numbers are 85 or 58.

Formula / Hints:
Two-digit expansion: Value $=$ 10(Tens) $+$ Units

Question 7*

Solution:
(i) $y = -3x + 4 \implies$ Slope $m = -3, y\text{-int} = (0, 4)$
(ii) $y = 2x + \frac{7}{2} \implies$ Slope $m = 2, y\text{-int} = (0, 3.5)$
(iii) $y = \frac{6}{5}x - 2 \implies$ Slope $m = \frac{6}{5}, y\text{-int} = (0, -2)$
(iv) $y = 2x - \frac{11}{3} \implies$ Slope $m = 2, y\text{-int} = (0, -\frac{11}{3})$
Lines (ii) and (iv) are parallel because slopes match.

Formula / Hints:
Slope-intercept form: $y = mx + c$.
Parallel condition: $m_1 = m_2$.

Question 8*

Solution:
(i) At $x = 313$: $y = \frac{9}{5}(313 - 273) + 32 = \frac{9}{5}(40) + 32 = 104°$F.
(ii) At $y = 158$: $158 = \frac{9}{5}(x - 273) + 32 \implies 126 = \frac{9}{5}(x - 273)$
$x - 273 = 126 \times \frac{5}{9} = 70 \implies x = 343$ K.

Formula / Hints:
Linear conversion evaluation.

Question 9*

Solution:
Equation: $w = 3d$.
At distance $d = 2$ units: Work done $w = 3(2) = 6$ units.

Formula / Hints:
Work (w) = Force $(3) \times$ Distance (d)

Question 10*

Solution:
(i) $m = \frac{11-5}{3-1} = 3 \implies y - 5 = 3(x - 1) \implies p(x) = 3x + 2$.
(ii) Cuts y-axis at $(0, 2)$. Cuts x-axis at $(-\frac{2}{3}, 0)$.

Formula / Hints:
Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$
Equation: $y - y_1 = m(x - x_1)$

Question 11*

Solution:
(i) $p(0) = 5 \implies b = 5$.
(iii) $p(x) + q(x) = (a+c)x + (5+d) = 6x + 4 \implies a + c = 6, d = -1$.
(ii) $p(3) - q(3) = 0 \implies (3a + 5) - (3c - 1) = 0 \implies a - c = -2$.
Solving system: $a = 2, c = 4$. Thus, $p(x) = 2x + 5$, $q(x) = 4x - 1$.

Hints:
Compare corresponding linear coefficients and constant targets.

Question 12*

Solution:
Stage 1: 6 sticks. Stage 2: 11 sticks. Stage 3: 16 sticks.
(i) Stage 4 = 21, Stage 5 = 26 sticks.
(iii) General stage formula: $5n + 1$.
(iv) Stage 15: $5(15) + 1 = 76$ sticks.
(v) $5n + 1 = 200 \implies 5n = 199 \implies n = 39.8$ (Not possible).

Formula / Hints:
Arithmetic term growth tracking pattern: $a_n = a_1 + (n - 1)d$.

Question 13*

Solution:
(i) $m_p = \frac{11-3}{6-2} = 2 \implies y - 3 = 2(x-2) \implies p(x) = 2x - 1$.
(iii) Parallel $\implies m_q = 2$. Passes $(4, -1) \implies q(x) = 2x - 9$.
X-intercepts ($y = 0$): For $p(x) \implies (0.5, 0)$; for $q(x) \implies (4.5, 0)$.

Formula / Hints:
Parallel slopes rule: $m_1 = m_2$.

Question 14

Solution:
All lines share a common x-intercept point at $(-1, 0)$.
Proof: Set $f(x) = 0 \implies ax + a = 0 \implies a(x+1) = 0 \implies x = -1$.

Hints:
Factor the expression to reveal invariant intercepts common across varying $a > 0$.
