Chapter 1: Real Numbers
Complete Board PYQs Notebook (Ex 1.3, 1.4, 1.5)

Exercise Set 1.3 (NCERT Core Selections)

Question 1 (v)

Solution:
Let $\frac{1}{\sqrt{5}} = \frac{p}{q}$ be rational ($\gcd(p, q) = 1, q \neq 0$).
Squaring: $\frac{1}{5} = \frac{p^2}{q^2} \implies q^2 = 5p^2$.
Since $5 \mid q^2$, $5 \mid q$. Let $q = 5k$.
$(5k)^2 = 5p^2 \implies 25k^2 = 5p^2 \implies p^2 = 5k^2$.
Since $5 \mid p^2$, $5 \mid p$. This contradicts $\gcd(p, q) = 1$.
Hence, $\frac{1}{\sqrt{5}}$ is irrational.

Formula / Hints:
Proof by contradiction method. If a prime $p \mid a^2$, then $p \mid a$.

Question 5

Solution:
If 6^n ends with 0, it must have 2 and 5 as prime factors.
$6^n = (2 \times 3)^n = 2^n \times 3^n$.
By Fundamental Theorem of Arithmetic, prime factors are unique. Since 5 is not present, 6^n cannot end with 0.

Hints:
A number ends with 0 if it contains factors 2×5.

Question 7

Solution:
From the bottom branches:
$w = 13 \times 7 = 91$.
$z = 3 \times w = 3 \times 91 = 273$.
$y = 2 \times 819 = 1638$.
$x = 2 \times y = 2 \times 1638 = 3276$.
Prime factorization of $x = 2^2 \times 3^2 \times 7 \times 13$.

Hints:
Multiply lower node factor paths up to the parent branch.

Question 8 (i)

Solution:
$2 \times 3 \times 5 \times 7 + 7 = 7(2 \times 3 \times 5 + 1) = 7 \times 31$.
Since the number has factors other than 1 and itself, it is a **composite number**. (True)

Hints:
Composite numbers have more than 2 distinct factors.

Question 8 (ii)

Solution:
$2 \times 3 \times 5 \times 7 + 1 = 210 + 1 = 211$.
Since 211 is a prime number, it cannot be composite. (False)

Hints:
Test for primality up to $\sqrt{211} \approx 14.5$.

Question 9

Solution:
$pqr + q = q(pr + 1)$.
Since q is a distinct prime, the expression has factors other than 1 and itself, making it a **composite number**.

Hints:
Factoring out a prime term reveals composite traits.

Exercise Set 1.4 (CBSE Board Questions)

Question 1 (iii)

Solution:
$96 = 2^5 \times 3, \quad 120 = 2^3 \times 3 \times 5.$
$\text{HCF} = 2^3 \times 3 = 24.$
$\text{LCM} = 2^5 \times 3 \times 5 = 480.$
Verification: $24 \times 480 = 11520 = 96 \times 120$. Verified.

Formula / Hints:
$\text{HCF} \times \text{LCM} = a \times b.$

Question 1 (iv)

Solution:
$72 = 2^3 \times 3^2, \quad 120 = 2^3 \times 3 \times 5.$
$\text{HCF} = 2^3 \times 3 = 24.$
$\text{LCM} = 2^3 \times 3^2 \times 5 = 360.$
Verification: $24 \times 360 = 8640 = 72 \times 120$. Verified.

Formula / Hints:
Product of symmetric power sets.

Question 2 (v)

Solution:
$26 = 2 \times 13, \quad 65 = 5 \times 13, \quad 117 = 3^2 \times 13.$
$\text{HCF} = 13.$
$\text{LCM} = 2 \times 5 \times 3^2 \times 13 = 1170.$

Hints:
Factor out lowest matching powers for HCF.

Question 4 (i)

Solution:
Given: $\text{HCF}(306, 657) = 9.$
$\text{LCM} = \frac{306 \times 657}{9} = 34 \times 657 = 22338.$

Formula / Hints:
$\text{LCM} = \frac{a \times b}{\text{HCF}}.$

Question 5

Solution:
If HCF = 16 and LCM = 380, HCF must divide LCM.
$\frac{380}{16} = 23.75$ (Not an integer).
Hence, this combination is **not possible**.

Hints:
HCF is always a factor of LCM.

Question 9

Solution:
Intervals: 6, 12, and 18 minutes.
$\text{LCM}(6, 12, 18) = 36$ minutes.
Next ring: 6:00 AM + 36 mins = 6:36 AM.

Hints:
Simultaneous occurrences require LCM evaluation.

Question 13

Solution:
Subtract remainders: $1251 - 1 = 1250$, $9377 - 2 = 9375$, $15628 - 3 = 15625$.
$1250 = 2 \times 5^4$, $9375 = 3 \times 5^5$, $15625 = 5^6$.
$\text{HCF} = 5^4 = 625$. Largest number is 625.

Hints:
Required values $= \text{HCF}(a - r_1, b - r_2, c - r_3)$.

Question 15

Solution:
Steps: 30 cm, 36 cm, 40 cm.
Minimum distance $= \text{LCM}(30, 36, 40) = 360$ cm.

Hints:
Find the least common multiple of step spans.

Question 16

Solution:
Intervals: 48s, 72s, 108s. LCM = 432 seconds.
432 seconds = 7 mins 12 seconds.
Next change: 7:07:12 AM.

Hints:
$\text{LCM} = 2^4 \times 3^3 = 432$.

Exercise Set 1.5 (All CBSE Board Questions)

Question 1 (b)

Solution:
Prove $7\sqrt{5}$ is irrational.
Let $7\sqrt{5} = r$ be rational $\implies \sqrt{5} = \frac{r}{7}$.
Since r is rational, $\frac{r}{7}$ is rational, contradicting that $\sqrt{5}$ is irrational.

Hints:
Rational $\times$ Irrational $=$ Irrational.

Question 1 (e)

Solution:
Prove $\frac{1}{\sqrt{5}}$ is irrational.
Identical system proof to Exercise 1.3 - Question 1(v).
Contradiction via $\gcd(p, q) = 1$ failure confirms irrationality.

Hints:
Board exam repetition variant mapping.

Question 2 (i)

Solution:
Prove $\frac{2}{\sqrt{7}}$ is irrational.
Let $\frac{2}{\sqrt{7}} = r$ be rational $\implies \sqrt{7} = \frac{2}{r}$.
Since r is rational, $\frac{2}{r}$ is rational, contradicting that $\sqrt{7}$ is irrational.

Hints:
Radical isolation transformation steps.

Question 2 (ii)

Solution:
Prove $\frac{3}{2\sqrt{5}}$ is irrational.
Let $\frac{3}{2\sqrt{5}} = r$ be rational $\implies \sqrt{5} = \frac{3}{2r}$.
Since r is rational, $\frac{3}{2r}$ is rational, contradicting that $\sqrt{5}$ is irrational.

Hints:
Transposition of scalar factors.

Question 2 (iii)

Solution:
Prove $6 + \sqrt{2}$ is irrational.
Let $6 + \sqrt{2} = r$ be rational $\implies \sqrt{2} = r - 6$.
Since r is rational, $r - 6$ is rational, contradicting that $\sqrt{2}$ is irrational.

Hints:
Rational $\pm$ Rational $=$ Rational.

Question 2 (iv)

Solution:
Prove $5\sqrt{2}$ is irrational.
Let $5\sqrt{2} = r$ be rational $\implies \sqrt{2} = \frac{r}{5}$.
Since r is rational, $\frac{r}{5}$ is rational, contradicting that $\sqrt{2}$ is irrational.

Hints:
Isolate the square root entity cleanly.

Question 8

Solution:
Prove $(5 + 3\sqrt{2})$ is irrational, given $\sqrt{2}$ is irrational.
Let $5 + 3\sqrt{2} = r$ be rational.
$3\sqrt{2} = r - 5 \implies \sqrt{2} = \frac{r-5}{3}$.
Since r is rational, $\frac{r-5}{3}$ is rational, contradicting that $\sqrt{2}$ is irrational.

Hints:
Combine linear algebraic transformations.

Question 9

Solution:
Prove $\frac{2+\sqrt{3}}{5}$ is irrational, given $\sqrt{3}$ is irrational.
Let $\frac{2+\sqrt{3}}{5} = r$ be rational.
$2 + \sqrt{3} = 5r \implies \sqrt{3} = 5r - 2$.
Since r is rational, $5r - 2$ is rational, contradicting that $\sqrt{3}$ is irrational.

Hints:
Isolate the radical expression directly.

Question 10

Solution:
Prove $2 + 5\sqrt{3}$ is irrational, given $\sqrt{3}$ is irrational.
Let $2 + 5\sqrt{3} = r$ be rational.
$5\sqrt{3} = r - 2 \implies \sqrt{3} = \frac{r-2}{5}$.
Since r is rational, $\frac{r-2}{5}$ is rational, contradicting that $\sqrt{3}$ is irrational.

Hints:
Multi-step scalar division steps.

Question 12

Solution:
Prove $2 + \sqrt{3}$ is irrational, given $\sqrt{3}$ is irrational.
Let $2 + \sqrt{3} = r$ be rational $\implies \sqrt{3} = r - 2$.
Since r is rational, $r - 2$ is rational, contradicting that $\sqrt{3}$ is irrational.

Hints:
Linear root transpositions.

Question 13

Solution:
Prove $5 - 2\sqrt{3}$ is irrational, given $\sqrt{3}$ is irrational.
Let $5 - 2\sqrt{3} = r$ be rational.
$2\sqrt{3} = 5 - r \implies \sqrt{3} = \frac{5-r}{2}$.
Since r is rational, $\frac{5-r}{2}$ is rational, contradicting that $\sqrt{3}$ is irrational.

Hints:
Maintain careful track of negative sign flips.

Question 14

Solution:
Prove $\frac{2-\sqrt{3}}{5}$ is irrational, given $\sqrt{3}$ is irrational.
Let $\frac{2-\sqrt{3}}{5} = r$ be rational.
$2 - \sqrt{3} = 5r \implies \sqrt{3} = 2 - 5r$.
Since r is rational, $2 - 5r$ is rational, contradicting that $\sqrt{3}$ is irrational.

Hints:
Clear denominators to check sets.

Question 15

Solution:
Prove $(\sqrt{2} + \sqrt{3})^2$ is irrational, given $\sqrt{6}$ is irrational.
Let $x = (\sqrt{2} + \sqrt{3})^2 = 2 + 2\sqrt{6} + 3 = 5 + 2\sqrt{6}$.
Assume x is rational $\implies 5 + 2\sqrt{6} = r$ (rational).
$2\sqrt{6} = r - 5 \implies \sqrt{6} = \frac{r-5}{2}$.
Since r is rational, $\frac{r-5}{2}$ is rational, contradicting that $\sqrt{6}$ is irrational.

Formula / Hints:
Expansion Identity: $(a + b)^2 = a^2 + 2ab + b^2$.

Question 16

Solution:
Prove $5\sqrt{3} + \frac{2}{3}$ is irrational, given $\sqrt{3}$ is irrational.
Let $5\sqrt{3} + \frac{2}{3} = r$ be rational.
$5\sqrt{3} = r - \frac{2}{3} \implies \sqrt{3} = \frac{3r-2}{15}$.
Since r is rational, $\frac{3r-2}{15}$ is rational, contradicting that $\sqrt{3}$ is irrational.

Hints:
Fraction subtraction across rational metrics.

Question 17

Solution:
Prove $4\sqrt{2} + \frac{5}{3}$ is irrational, given $\sqrt{2}$ is irrational.
Let $4\sqrt{2} + \frac{5}{3} = r$ be rational.
$4\sqrt{2} = r - \frac{5}{3} \implies \sqrt{2} = \frac{3r-5}{12}$.
Since r is rational, $\frac{3r-5}{12}$ is rational, contradicting that $\sqrt{2}$ is irrational.

Hints:
Isolate radical components methodically.

Question 18

Solution:
Prove $\sqrt{p}$ is irrational for any prime positive integer p.
Let $\sqrt{p} = \frac{a}{b}$ $(\gcd(a, b) = 1, b \neq 0) \implies a^2 = pb^2$.
Since $p \mid a^2$, $p \mid a \implies a = pk$.
$(pk)^2 = pb^2 \implies p^2k^2 = pb^2 \implies b^2 = pk^2$.
Since $p \mid b^2$, $p \mid b$. This contradicts $\gcd(a, b) = 1$.

Hints:
Generalized standard radical proof for prime integer bases.